IN FERMI–LIQUID MODEL

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Abstract

Influence of asymmetry on superfluidity of nuclear matter with triplet–singlet pairing of nucleons (in spin and isospin spaces) is considered within the framework of a Fermi–liquid theory. Solutions of self–consistent equations for the critical temperature and the energy gap at $T = 0$ are obtained, which under specific values of density and the asymmetry parameter α of nuclear matter demonstrate double–valued behaviour. At $T = 0$ the solution with the gap, being very close on value to the energy gap at $\alpha = 0$, is thermodynamically favorable. It is shown, that such solution appears as a result of the first order phase transition in density.

PACS: 21.65.+f; 21.30.Fe; 71.10.Ay
Keywords: asymmetric nuclear matter, neutron–proton pairing, critical temperature, energy gap, Fermi–liquid theory, Skyrme forces.

Introduction. It is well established, that neutron–proton (np) pairing plays an essential role in description of superfluidity of finite nuclei with $N = Z$ (see Ref. [1] and references therein) and symmetric nuclear matter [2]–[4]. In this report we shall investigate influence of asymmetry on np superfluidity of nuclear matter. Earlier this problem was treated with the use of various approaches and potentials of NN interaction. In particular, the cases of ${}^3S_1$–${}^3D_1$ and ${}^3D_2$ pairing were considered in Refs. [2, 5] on the base of the Thouless criterion for the thermodynamic T-matrix. As a potential of NN interaction, there where chosen the Graz II and Paris potential, respectively. Superfluidity in ${}^3S_1$–${}^3D_1$ pairing channel was studied also in Ref. [6] within BCS theory of superconductivity with the use of the Paris potential in the separable form. Investigations, based on the Thouless criterion, deduce the suppression of np pairing correlations with increase of isospin asymmetry. However, the Thouless criterion can be exploited for finding the critical temperature only, but does not permit to draw any conclusions about superfluidity with a finite gap. The studies in Ref. [6], based on the BCS theory, were carried out with the use of the bare interaction and the single particle spectrum of a free Fermi gas and give, thus, overestimated values of the energy gap. The effect of ladder–renormalized single particle spectrum [7] on the magnitude of the energy gap was investigated in Ref. [8]. In this work np pairing in ${}^3S_1$–${}^3D_1$ pairing channel was considered within BCS theory with the single particle energy, taken from the extended Brueckner–Hartree–Fock (BHF) approach. The Argonne V_{14} potential was explored as input for determination of the single particle energy and the bare interaction in the form of the Paris potential was used to evaluate the energy gap. The use of the bare interaction in the gap equation might appear to be a very strong simplification, because medium polarization strongly reduces the magnitude of the gap (see Ref. [9] for the influence of the polarization effects on the pairing force in the ${}^1S_0$ channel). In principle, the effective pairing interaction should be obtained by means of Brueckner renormalization, which gives the correct interaction after modifying the bare interaction for the effect of nuclear medium. However, the issue of microscopic many–body calculation of the effective pairing potential is a complex one and still is not solved. For this reason, it is quite natural step to develop some kind of a phenomenological theory, where instead of microscopical calculation of the pairing interaction one exploits the phenomenological effective interaction. We shall investigate influence of asymmetry on superfluid properties of nuclear matter, using Landau's semiphenomenological theory of a Fermi–liquid (FL). In the Fermi–liquid model the normal and anomalous FL interaction amplitudes are taken into account on the equal footing. This will allow us to consider consistently influence of the normal FL amplitudes on superfluid properties of nuclear matter. Besides, as a potential of NN interaction we choose the Skyrme effective forces, describing interaction of two nucleons in the presence of nucleon medium. The Skyrme forces are widely used in description of nuclear system properties and, in particular, they were exploited for description of superfluid properties of finite nuclei [10, 11] as well as infinite symmetric nuclear matter [12]-[15].

1. Basic equations. The basic formalism is laid out in more detail in Ref. [14], where superfluidity of symmetric nuclear matter was studied. As was shown there, superfluidity with triplet–singlet (TS) pairing of nucleons (total spin S and isospin T of a pair are equal $S=1$, $T=0$) is realized near the saturation density in symmetric nuclear matter with the Skyrme interaction. Therefore, further we shall study influence of asymmetry on superfluid properties of TS phase of nuclear matter. For the states with the projections of total spin and isospin $S_z=T_z=0$ the normal distribution function f and the anomalous distribution function g have the form

$$f(\vec{p})=f_{00}(\vec{p})\sigma_0\tau_0+f_{03}(\vec{p})\sigma_0\tau_3,\quad g(\vec{p})=g_{30}(\vec{p})\sigma_3\sigma_2\tau_2 \tag{1}$$

where σ_i,τ_k are the Pauli matrices in spin and isospin spaces. The operator of quasiparticle energy ε and the matrix order parameter Δ of the system for the energy functional E, being invariant with respect to rotations in spin and isospin spaces, have the analogous structure:

$$\varepsilon(\vec{p})=\varepsilon_{00}(\vec{p})\sigma_0\tau_0+\varepsilon_{03}(\vec{p})\sigma_0\tau_3,\quad \Delta(\vec{p})=\Delta_{30}(\vec{p})\sigma_3\sigma_2\tau_2. \tag{2}$$

The matrix self-consistent equation for determining the distribution functions f and g follows from the minimum condition for the thermodynamic potential

$$\Omega=-S+Y_0E+Y_{4a}N_a+Y_{4b}N_b \tag{3}$$

(S is entropy, N_a,N_b are the numbers of particles of species a and b, $Y_0=1/T$, $Y_{4a}=-\mu_a^0/T$ and $Y_{4b}=-\mu_b^0/T$ are the Lagrange multipliers, μ_a^0 and μ_b^0 are the chemical potentials of particles of species a and b, T is temperature).

Using the procedure of block diagonalization [16] of the matrix self–consistent equation, one can express evidently the distribution functions f_{00}, f_{03} and g_{30} in terms of the quantities ε and Δ:

$$\begin{aligned} f_{00}&=\frac{1}{2}-\frac{\xi_{00}}{4E}\Bigl(\tanh\frac{Y_0(E+\xi_{03})}{2}+\tanh\frac{Y_0(E-\xi_{03})}{2}\Bigr),\\ f_{03}&=-\frac{\xi_{00}}{4E}\Bigl(\tanh\frac{Y_0(E+\xi_{03})}{2}-\tanh\frac{Y_0(E-\xi_{03})}{2}\Bigr),\\ g_{30}&=-\frac{\Delta_{30}}{4E}\Bigl(\tanh\frac{Y_0(E+\xi_{03})}{2}+\tanh\frac{Y_0(E-\xi_{03})}{2}\Bigr)\end{aligned} \tag{4}$$

Here

$$E=\sqrt{\xi_{00}^2+\Delta_{30}^2},\ \xi_{00}=\varepsilon_{00}-\mu_{00}^0,\ \xi_{03}=\varepsilon_{03}-\mu_{03}^0,\ \mu_{00}^0=\frac{\mu_p^0+\mu_n^0}{2},\ \mu_{03}^0=\frac{\mu_p^0-\mu_n^0}{2}$$

To obtain the closed system of equations for the quasiparticle energy ε and the energy gap Δ, it is necessary to express the quantities ε,Δ through the distribution functions f and g. For this purpose one has to set the energy functional $E(f,g)$ of the system.

Renormalization of nucleon masses and chemical potentials. For asymmetric nuclear matter with TS pairing of nucleons we set the energy functional in the form

$$E(f,g)=E_0(f)+E_{int}(f)+E_{int}(g),\qquad E_0(f)=\frac{4}{\mathcal{V}}\sum_{\vec p}\varepsilon_0^0(\vec p)f_{00}(\vec p),\tag{5}$$

$$E_{int}(f)=2\sum_{\vec p}\left(\tilde\varepsilon_{00}(\vec p)f_{00}(\vec p)+\tilde\varepsilon_{03}(\vec p)f_{03}(\vec p)\right),\ E_{int}(g)=\frac{2}{\mathcal{V}}\sum_{\vec p,\vec q}g_{30}^*(\vec p)V_1(\vec p,\vec q)g_{30}(\vec q),$$

where

$$\tilde\varepsilon_{00}(\vec p)=\frac{1}{2\mathcal{V}}\sum_{\vec q}U_0(\vec k)f_{00}(\vec q),\ \tilde\varepsilon_{03}(\vec p)=\frac{1}{2\mathcal{V}}\sum_{\vec q}U_2(\vec k)f_{03}(\vec q),\ \vec k=\frac{\vec p-\vec q}{2}\tag{6}$$

Here $\varepsilon_0^0(\vec p)=\frac{\vec p^{\,2}}{2m_{00}^0}$ is the kinetic energy of a free nucleon; $U_0(\vec k),U_2(\vec k)$ are the normal FL amplitudes, $V_1(\vec p,\vec q)$ is the anomalous FL amplitude. Differentiating the energy functional with respect to normal and anomalous distribution functions [16], we obtain the equations for the quantities ξ and Δ, entering into Eqs. (4):

$$\xi_{00}(\vec p)=\varepsilon_0^0(\vec p)-\mu_{00}^0+\tilde\varepsilon_{00}(\vec p),\ \xi_{03}(\vec p)=\tilde\varepsilon_{03}(\vec p)-\mu_{03}^0,\tag{7}$$

$$\Delta_{30}(\vec p)=\frac{1}{\mathcal{V}}\sum_{\vec q}V_1(\vec p,\vec q)g_{30}(\vec q)$$

In view of Eq. (4), the equation for the energy gap takes the form

$$\Delta_{30}(\vec p)=-\frac{1}{4\mathcal{V}}\sum_{\vec q}V_1(\vec p,\vec q)\frac{\Delta_{30}(\vec q)}{E(\vec q)}\Big\{\tanh\frac{Y_0(E(\vec q)+\xi_{03}(\vec q))}{2}+\tanh\frac{Y_0(E(\vec q)-\xi_{03}(\vec q))}{2}\Big\}\tag{8}$$

Later on for obtaining numerical results we shall use the effective Skyrme interaction as a potential of NN interaction [17]. The expressions for the normal amplitudes U_0,U_2 and the anomalous amplitude V_1 in the case of the Skyrme interaction have the form [14]:

$$U_0(\vec k)=6t_0+6t_3\varrho^\beta+\frac{1}{\hbar^2}[6t_1+2t_2(5+4x_2)]\vec k^{\,2},\tag{9}$$

$$U_2(\vec k)=-2t_0(1+2x_0)-\frac{1}{3}t_3\varrho^\beta(1+2x_3)-\frac{2}{\hbar^2}[t_1(1+2x_1)-t_2(1+2x_2)]\vec k^{\,2},$$

$$V_1(\vec p,\vec q)=t_0(1+x_0)+\frac{1}{6}t_3\varrho^\beta(1+x_3)+\frac{1}{2\hbar^2}t_1(1+x_1)(\vec p^{\,2}+\vec q^{\,2}),$$

where ϱ is density of asymmetric nuclear matter, t_i,x_i,β are some phenomenological parameters. There are sets of parameters t_i,x_i,β, which differ for various versions of the Skyrme forces (we shall use the SkP [10], SkX [18] and SkSC4 [19] potentials). Note, that account of the normal FL amplitudes in the case of the effective Skyrme interaction, being quadratic on momenta, is reduced to renormalization of free nucleon masses and chemical potentials. Really, substituting the expressions for the amplitudes $U_0(\vec{k}), U_2(\vec{k})$ into Eq. (7), we obtain

$$\xi_{00} = \frac{p^2}{2m_{00}} - \mu_{00}, \quad \xi_{03} = \frac{p^2}{2m_{03}} - \mu_{03}, \tag{10}$$

where the renormalized mass m_{00} of a nucleon and the effective isovector mass m_{03} are defined by the formulae

$$\frac{\hbar^2}{2m_{00}} = \frac{\hbar^2}{2m_{00}^0} + \frac{\varrho}{16}[3t_1 + t_2(5+4x_2)], \quad \frac{\hbar^2}{2m_{03}} = \frac{\alpha\varrho}{16}[t_1(1+2x_1) - t_2(1+2x_2)] \tag{11}$$

Here $\alpha = (\varrho_n - \varrho_p)/\varrho$ is the asymmetry parameter of nuclear matter, ϱ_n and ϱ_p are the partial number densities of neutrons and protons. The renormalized chemical potentials μ_{00}, μ_{03} are defined from the normalization conditions

$$\frac{4}{\mathcal{V}}\sum_{\vec{p}} f_{00}(\vec{p}) = \varrho_p + \varrho_n, \quad \frac{4}{\mathcal{V}}\sum_{\vec{p}} f_{03}(\vec{p}) = \varrho_p - \varrho_n \tag{12}$$

and in the leading approximation on the ratios $T/\varepsilon_F, \Delta/\varepsilon_F$ have the form

$$\mu_{00} = \frac{1}{2}(\mu_p + \mu_n), \ \ \mu_{03} = \frac{1}{2}(\mu_p - \mu_n), \quad \mu_{p,n} = \frac{\hbar^2 k_{F_{p,n}}^2}{2m_{p,n}}, \tag{13}$$

where $k_{F_{p,n}} = (3\pi^2\varrho_{p,n})^{1/3}$. The proton and neutron effective masses, defined by the formulae

$$\frac{2}{m_{00}} = \frac{1}{m_p} + \frac{1}{m_n}, \quad \frac{2}{m_{03}} = \frac{1}{m_p} - \frac{1}{m_n},$$

according to Eq. (11), can be written as

$$m_p = \frac{m_{00}}{1+\alpha\varrho r}, \quad m_n = \frac{m_{00}}{1-\alpha\varrho r}, \quad r = \frac{m_{00}^0[t_1(1+2x_1) - t_2(1+2x_2)]}{8\hbar^2 + \varrho m_{00}^0[3t_1 + t_2(5+4x_2)]} \tag{14}$$

In Eqs. (14) we explicitly single out the dependence from the asymmetry parameter α. With account of Eqs. (12), (13), for the renormalized chemical potentials μ_{00}, μ_{03} we obtain

$$\mu_{00} = \frac{\mu_0}{2}\{(1-\alpha)^{\frac{2}{3}}(1+\alpha\varrho r) + (1+\alpha)^{\frac{2}{3}}(1-\alpha\varrho r)\}, \quad \mu_0 \equiv \frac{\hbar^2}{2m_{00}}\Bigl(\frac{3\pi^2\varrho}{2}\Bigr)^{2/3} \tag{15}$$

$$\mu_{03} = \frac{\mu_0}{2}\{(1-\alpha)^{\frac{2}{3}}(1+\alpha\varrho r) - (1+\alpha)^{\frac{2}{3}}(1-\alpha\varrho r)\}$$

Critical temperature. The critical temperature of transition to TS superfluid phase is found from Eq. (8), determining the energy gap, in the linear on Δ approximation

$$\Delta_{30}(\vec{p}) = -\frac{1}{4\mathcal{V}}\sum_{\vec{q}} V_1(\vec{p},\vec{q})\frac{\Delta_{30}(\vec{q})}{\xi_{00}(\vec{q})}\Bigl\{\tanh\frac{Y_0(\xi_{00}(\vec{q}) + \xi_{03}(\vec{q}))}{2} + \tanh\frac{Y_0(\xi_{00}(\vec{q}) - \xi_{03}(\vec{q}))}{2}\Bigr\} \tag{16}$$

Considering, that the interaction amplitude V_1 is not equal to zero only in a narrow layer near the Fermi–surface, $|\xi_{00}| \le \theta$ (in numerical calculations we set $\theta = 0.1\mu_0$), we present Eq. (16) in the form

$$1 = g\int_{-\theta}^{\theta}\frac{d\xi_{00}}{\xi_{00}}\Bigl\{\tanh\frac{Y_0(\xi_{00}[1+\frac{m_{00}}{m_{03}}]+\psi)}{2}+\tanh\frac{Y_0(\xi_{00}[1-\frac{m_{00}}{m_{03}}]-\psi)}{2}\Bigr\}, \qquad (17)$$

$$g=-\frac{\nu_F V_1(p=p_F,q=p_F)}{4},\quad \psi=\frac{m_{00}}{m_{03}}\mu_{00}-\mu_{03},\quad \nu_F=\frac{m_{00}p_F}{2\pi^2\hbar^3},\quad p_F=\sqrt{2m_{00}\mu_{00}}$$

The results of numerical integration of Eq. (17) are presented in Fig. 1. In the case of symmetric nuclear matter ($\alpha = 0$) we obtain the phase curve with single–valued behavior of the transition temperature. For small values of asymmetry α there exist such regions of large and low densities of nuclear matter, for which we have two critical temperatures (note, that at $\alpha \neq 0$ Eq. (17) has no solutions for $n = 0$ and directly near by $n = 0$). When α increases, these regions begin to approach and at some value $\alpha = \alpha_c$ ($\alpha_c \approx 0.07 \div 0.08$) there takes place contiguity of the regions, so that we have always only two critical temperatures (for the regions, where solutions exist). For $\alpha > \alpha_c$ the phase curves come off the density axis and turn into the closed oval curves. Under further increasing of α the dimension of the oval curves is reduced and at some $\alpha = \alpha_m$ the oval curves contract to a point. For the values $\alpha > \alpha_m$ the triplet–singlet superfluidity fails. The value of α_m ranges approximately from 0.09 to 0.11. Note, that since Eq. (17) is obtained in the limit $\Delta \to 0$, the phase transitions, considering here, are the phase transitions of the second order.

The appearance of two critical temperatures in asymmetric nuclear matter was found in Ref. [5] for the case of $^3D_2$ pairing of nucleons with the use of the Paris NN potential, taken as the bare interaction in the equation for T_c. Comparing obtained results, one can conclude, that despite of distinction between the pairing channels and pairing forces, considered in our work and in Ref. [5], the qualitative behaviour of the phase curves is similar and the conclusion about impossibility of TS superfluidity is valid at large values of α. In our case we have more higher values of the threshold asymmetry α_m and the critical temperature (in Ref. [5] $\alpha_m \approx 0.02$ and $\max T_c \approx 0.6$ MeV), but the density region, where TS superfluidity exists, extends to much smaller densities $0.17 \div 0.24$ fm^{-3} (depending on the type of Skyrme forces), instead of bounding density $\simeq 1.2$ fm^{-3} in Ref. [5]. Note also, that from two solutions with different critical temperatures that solution will be actually realized, to which corresponds smaller thermodynamic potential.

Energy gap at $T = 0$. As follows from Eq. (8), equation, determining the energy gap at $T = 0$, has the form:

$$\Delta_{30}(\vec p) = -\frac{1}{4\mathcal{V}}\sum_{\vec q} V_1(\vec p,\vec q)\frac{\Delta_{30}(\vec q)}{E(\vec q)}\left\{\mathrm{sgn}\,(E(\vec q)+\xi_{03}(\vec q))+\mathrm{sgn}\,(E(\vec q)-\xi_{03}(\vec q))\right\} \qquad (18)$$

Passing in Eq. (18) to integration on a layer and entering new variables $x = \xi_{00}/\mu_0$, $\theta_0 = \theta/\mu_0$, $y = \Delta_{30}(p = p_F)/\mu_0$, we arrive at the equation for determining the dimensionless gap y:

$$1 = g\int_{-\theta_0}^{\theta_0}\frac{dx}{\sqrt{x^2+y^2}}\Bigl\{\mathrm{sgn}\,\bigl(\sqrt{x^2+y^2}+\frac{m_{00}}{m_{03}}(x+\frac{\mu_{00}}{\mu_0})-\frac{\mu_{03}}{\mu_0}\bigr)+\mathrm{sgn}\,\bigl(\sqrt{x^2+y^2}-\frac{m_{00}}{m_{03}}(x+\frac{\mu_{00}}{\mu_0})+\frac{\mu_{03}}{\mu_0}\bigr)\Bigr\} \qquad (19)$$

The contribution to the integral gives the range of values x, for which the functions, standing as arguments of the functions sgn, have the same sign (namely, positive). If to designate these functions as $f_1(x,y)$ and $f_2(x,y)$, respectively, and to enter a function $F(x,y) = f_1(x,y)f_2(x,y)$, the contribution to the integral gives the domain on x, for which $\operatorname{sgn} F(x,y) = 1$. Since $F(x,y)$ is a quadratic function of x, one can find easily the boundary points x_-, x_+ $(x_+ > x_-)$, separating the regions with $\operatorname{sgn} F(x,y) = 1$ and $\operatorname{sgn} F(x,y) = -1$. In particular, such values of the gap, density and asymmetry parameter of nuclear matter are possible, that the function $F(x,y)$ has no roots with respect to x. In this case the whole domain from $-\theta_0$ to θ_0 gives the contribution to the integral in Eq. (19) and we arrive at the equation of the BCS type at $T = 0$:

$$1 = 2g \ln \frac{\theta_0 + \sqrt{\theta_0^2 + y^2}}{-\theta_0 + \sqrt{\theta_0^2 + y^2}} \tag{20}$$

with the solution $y = \theta_0/(\sinh(1/4g)) \equiv y_{BCS}$. Here the TS coupling constant g depends not only from density, but also from the asymmetry parameter (see Eqs. (15), (17)). However, for small values of asymmetry (when TS superfluidity exists), the dependence on α is weak. In general case, the dependence on α is contained mainly through the dependence on α of the boundary points $x_-(\alpha), x_+(\alpha)$, which determine the admissible domain of integration with $\operatorname{sgn} F = 1$. Note, that we can arrive also at Eq. (20), if $x_- > \theta_0$ (from an explicit form of the root x_+ it follows, that, since for $\alpha > 0$ we have $m_{03} > 0, \mu_{03} < 0$, the case $x_+ < -\theta_0$ is impossible). However, our calculations show that in fact this case is not realized.

The results of numerical integration of Eq. (19) are presented in Fig. 2. In the case of symmetric nuclear matter ($\alpha = 0$) we obtain the phase curve with one–valued behavior of the gap. For small values of asymmetry α there exist such regions of large and low densities of nuclear matter (excluding some vicinity of the point $n = 0$), for which we have two values of the energy gap, where one of these values is the solution of Eq. (20) and practically coincides with the corresponding value of the gap for the case $\alpha = 0$ (the difference in the values of the gaps is of the order 10^{-3}MeV and less, so that corresponding parts of the phase curves are practically indistinguishable). When α increases, these regions begin to approach and at some value $\alpha = \alpha_c$ (the same as for the phase curves $T_c(\varrho)$) there takes place contiguity of the regions with two solutions. For $\alpha > \alpha_c$ two branches of the phase curves come off the density axis and combine to one curve, beginning and ending in some points of the phase curve with $\alpha = 0$. When α increases further, these points move towards and at some $\alpha = \alpha'_m$ (not equal to α_m for the phase curves $T_c(\varrho)$) the branches of solutions contract to a point. The value α'_m determines the maximum value of the asymmetry parameter, when TS superfluidity exists at $T = 0$ ($\alpha'_m \approx 0.15 \div 0.16$).

The temperature dependence of the energy gap was studied in Refs. [6, 8] with the use of the bare interaction in the gap equation in the form of Paris and Argonne V_{14} potential, respectively. Comparing obtained results at $T = 0$, one concludes, that there is a qualitative difference in behavior of the energy gap: in our case for some values of density and asymmetry parameter the gap demonstrates double–valued behavior instead of universal single–valued behavior in Refs. [6, 8]. This point is one of the main results of this work. As clear from previous consideration, if at some fixed value of density and enough small asymmetry α, $0 < \alpha < \alpha_c$, the gap has a unique value $y = y_{BCS}$, then under further increasing of asymmetry, for the values $\alpha_c \le \alpha \le \alpha'_m$, the gap necessarily has two values: one is $y = y_{BCS}$ and the second is something less. In order to confirm our numerical results, in Appendix we present results of analytical consideration in the framework of cut–off interaction model. As follows from that analysis, the double–valued behavior of the energy gap is preserved also for the case, when we use the bare, density–independent interaction instead of the effective interaction.

Note, that our calculations of the energy gap and critical temperature within FL approach are fully self-consistent. In particular, with sufficient accuracy the inequalities $\Delta_{30}(p = p_F)/\varepsilon_F \ll 1$, $T_c/\varepsilon_F \ll 1$ ($\varepsilon_F = \mu_{00}$) are fulfilled for all densities, where superfluidity exists. For example, the maximum values of the ratio $\Delta_{30}(p = p_F)/\varepsilon_F$ are equal 0.122, 0.115 and 0.149 for the SkP, SkX and SkSC4 potentials, respectively, at corresponding densities 0.025, 0.045 and 0.033 fm^{-3}. For the critical temperature an agreement is still better. Therefore, our approximation, consisting in disregard by influence of the finite size of the gap on the single particle energy and chemical potentials, is quite justified. The important point here is the made assumption, that integration in Eqs. (17), (19) is performed on a narrow layer near the Fermi surface. The value of the cut–off parameter θ is determined only from the requirement $\theta \ll \varepsilon_F$. If integration would be performed on all admissible area of momentum space, we, generally speaking, would obtain in the case of strong interaction the critical temperature and the energy gap, being comparable with ε_F. However, this would lead out us for the limits of applicability of a FL theory. In this connection it should be pointed out the analysis of Ref. [20], accomplished for the case of symmetric nuclear matter at zero temperature. The results of this work, obtained by combined, self-consistent treatment of the BCS gap equation and the BHF equations with account of the finite size of the gap, show substantial modification of the local effective mass close to the Fermi surface at low density comparing to the case of uncoupled calculation. Hence, a FL description fails probably at low–density limit.

Thermodynamic stability. In the case, when at given density and asymmetry we have a few solutions with different values of the energy gap, it is necessary to compare the thermodynamic potentials of these solutions to clarify which solution is thermodynamically favorable. According to Eq. (3), the expression for the potential $\Omega' = \Omega/Y_0$ at $T = 0$ can be written in the form

$$\Omega' = \mathrm{Tr}_\kappa \xi f - 2\sum_{\vec{p}}(\tilde{\varepsilon}_{00}(\vec{p})f_{00}(\vec{p}) + \tilde{\varepsilon}_{03}(\vec{p})f_{03}(\vec{p})) - \frac{1}{2}\sum_{\vec{p}}\frac{\Delta_p^2}{E_p}\{\mathrm{sgn}(E_p + \xi_{03}(\vec{p})) + \mathrm{sgn}(E_p - \xi_{03}(\vec{p}))\}, \tag{21}$$

$\kappa \equiv (\vec{p}, \sigma, \tau)$, σ and τ are the projections of nucleon spin and isospin. The results of numerical determination of the thermodynamic potential density $\omega = \Omega'/\mathcal{V}$, measured from the potential ω_n of the normal state, for various solutions of the self–consistent Eq. (19) are presented in Fig. 3. Qualitatively the results for different versions of the Skyrme interaction coincide. Let us consider, for example, the case of SKP potential. The solid curve corresponds to symmetric nuclear matter ($\alpha = 0$). At $\alpha = 0.07$ we have three branches of solutions, corresponding curves are plotted by dashed line. The smallest potential has the solution with greatest Δ, i.e., with the gap, being very close on value to the gap at $\alpha = 0$ (see corresponding part of the dome-shaped curve in Fig. 2). At $\alpha = 0.09$ we have two branches (dash–dotted lines), where the branch, situated on the dome–shaped curve in Fig. 2, is again thermodynamically favorable. It follows from this, that, since for $\alpha \neq 0$ the solutions, corresponding to the part of dome–shaped curve, arise at density change by a jump, then all superfluid phase transitions in asymmetric nuclear matter, considered in the previous section (at $T = 0$), are the first order phase transitions in density.

In principle, this phase transition could be observed under study of intermediate heavy ion reactions. If to assume, that the final stage of the reaction can be described as an expansion of a compound nucleon system, formed in heavy ion collision, then under lowering density this disassembling phase can undergo a first order phase transition in density from the normal to superfluid state.

Conclusion. Thus, we have considered influence of asymmetry on TS superfluidity of nuclear matter with the Skyrme interaction. The study is done on the base of a Fermi–liquid theory, in which the normal and anomalous interaction amplitudes are taken into account on a par. Account of normal interaction amplitudes leads to renormalization of masses of a proton and a neutron and to renormalization of the chemical potentials.

We have obtained solutions of the self–consistent equation, determining the critical temperature of the second order phase transition to TS superfluid phase. It is shown, that for the values of asymmetry $0 < \alpha < \alpha_c$ ($\alpha_c \approx 0.07 \div 0.08$) the equation for the transition temperature has one or two solutions (that depends on value of density of nuclear matter) and for $\alpha_c \le \alpha \le \alpha_m$ it has only two solutions ($\alpha_m \approx 0.09 \div 0.11$). At $\alpha > \alpha_m$ TS superfluidity of nuclear matter disappears.

We have done analysis of the self–consistent equation for determining the energy gap at $T = 0$. For the values of asymmetry $0 < \alpha < \alpha_c$ we have one or two solutions of the self–consistent equation, for $\alpha_c \le \alpha \le \alpha'_m$ we have only two solutions ($\alpha'_m \approx 0.15 \div 0.16$). Analysis of thermodynamic stability shows, that the solution with the gap, being very close on value to the gap at $\alpha = 0$, is thermodynamically favorable. Since at $\alpha \neq 0$ the gap for this solution is everywhere finite, the superfluid transition to the given phase is the phase transition of the first order in density.

Appendix

Here we analytically study the behavior of the energy gap as a function of density and asymmetry parameter at $T = 0$ in the limit of small asymmetry, $\alpha \ll 1$. In this case up to the terms of the order α^2 the Eq. (19) can be presented in the form

$$1 = 2g \int_0^{\theta_0} \frac{dx}{\sqrt{x^2 + y^2}} \Big\{1 + \mathrm{sgn}(x^2 - x_+^2)\Big\}, \tag{22}$$

where

$$x_+^2 = y_k^2 - y^2, \quad y_k(\alpha) = \frac{2}{3}\alpha$$

Then for $y > y_k$ we have the solution $y = y_{BCS}$ and for $y < y_k$ the solution is

$$y^2 = y_k^2 \Big\{1 - \Big[1 - \frac{2h}{1 + h^2}(h - \frac{\theta_0}{y_k})\Big]^2\Big\}, \quad h \equiv \exp(\frac{1}{4g}) \tag{23}$$

The condition $y_k = y_{BCS}$ determines the branching points with respect to ϱ (at fixed α, such that $0 < \alpha < \alpha_c$): $\varrho = \varrho_1(\alpha)$, $\varrho = \varrho_2(\alpha)$. In these points the solutions $y = y_1(\varrho, \alpha)$ and $y = y_2(\varrho, \alpha)$ branch off the solution $y = y_{BCS}$. These solutions vanish at some points $\varrho'_1(\alpha)$, $\varrho'_2(\alpha)$. The critical point α_c is determined from the condition $\varrho'_1(\alpha) = \varrho'_2(\alpha)$. For small values of asymmetry the quantity α_c is given by the formula $\alpha_c = \frac{3}{2}\theta_0 \exp(-1/(4g_{max}))$, where $g_{max} = \max g(\varrho, \alpha = 0)$. For $\alpha > \alpha_c$ the branches $y_1(\varrho, \alpha)$ and $y_2(\varrho, \alpha)$ are connected to one curve, described by Eq. (23). When α increases further, this curve and the curve

$y = y_{BCS}$ begin to shrink and contract to a point at some values $\varrho = \varrho'_m, \alpha = \alpha'_m$, which are determined by the equations $y_k(\alpha) = y_{BCS}(\varrho, \alpha)$, $\frac{\partial y_{BCS}(\varrho,\alpha)}{\partial \varrho} = 0$. At $\alpha > \alpha_m$ the Eq. (22) has no solutions with finite y, that corresponds to the absence of TS superfluidity of asymmetric nuclear matter.

Note, that given above analysis is correct, if we introduce a cut–off in the integral in the gap equation (18) with the cut–off parameter θ under quite general assumptions on behavior of the pairing interaction, namely, that the coupling constant g should tend to zero at $\varrho \to 0$ and $\varrho \to \infty$. Qualitatively the behavior of the energy gap as a function of density and asymmetry parameter at $T = 0$ is preserved also for the case, when the pairing potential $V_1(\vec{p}, \vec{q})$ and the cut–off parameter θ do not depend on density (the kernel in the gap equation has the form of the bare interaction). In this case it is important, that dimensionless cut–off parameter $\theta_0 = \theta/\mu_0$ behaves as $\varrho^{-2/3}$ at $\varrho \to \infty$.

Acknowledgement. Authors thank BMBF for financial support of this work and acknowledge information support of INTAS under grant # 98–01.

References

[1] A.L. Goodman, Phys. Rev. C **60** (1999) 014311.

[2] Th. Alm, B.L. Friman, G. Röpke et al., Nucl. Phys. A **551** (1993) 45.

[3] M. Baldo, U. Lombardo and P. Schuck, Phys. Rev. C **52** (1995) 975.

[4] E. Garrido, P. Sarriguren, E. Moya de Guerra et al., Phys. Rev. C **60** (1999) 064312.

[5] Th. Alm, G. Röpke, A. Sedrakian et al., Nucl. Phys. A **604** (1996) 491.

[6] A. Sedrakian, T. Alm and U. Lombardo, Phys. Rev. C **55** (1997) R582.

[7] W. Zuo, I. Bombaci and U. Lombardo, Phys. Rev. C **60** (1999) 024605.

[8] A. Sedrakian and U. Lombardo, Phys. Rev. Lett. **84** (2000) 602.

[9] H.-J. Schulze, J. Cugnon, A. Lejeune et al., Phys. Lett. B **375** (1996) 1.

[10] J. Dobaczewski, H. Flocard and J. Treiner, Nucl. Phys. A **422** (1984) 103.

[11] P.-G. Reinhard, D.J. Dean, W. Nazarewicz et al., Phys. Rev. C **60** (1999) 014316.

[12] R.K. Su, S.D. Yang and T.T.S. Kuo, Phys. Rev. C **35** (1987) 1539.

[13] M.F. Jiang and T.T.S. Kuo, Nucl. Phys. A **481** (1988) 294.

[14] A.I. Akhiezer, A.A. Isayev, S.V. Peletminsky et al., Zh. Eksp. Teor. Fiz. **112** (1997) 3 [Sov. Phys. JETP **85** (1997) 1].

[15] A.I. Akhiezer, A.A. Isayev, S.V. Peletminsky et al., Phys. Lett. B **451** (1999) 430.

[16] A.I. Akhiezer, V.V. Krasil'nikov, S.V. Peletminsky et al., Phys. Rep. **245** (1994) 1.

[17] M. Brack, C. Guet and H.-B. Hakansson, Phys. Rep. **123** (1985) 275.

[18] B.A. Brown, Phys. Rev. C **58** (1998) 220.

[19] Y. Aboussir, J.M. Pearson, A.K. Dutta et al., Nucl. Phys. A **549** (1992) 155.

[20] U. Lombardo, H.-J. Schulze and W. Zuo, Phys. Rev. C **59** (1999) 2927.

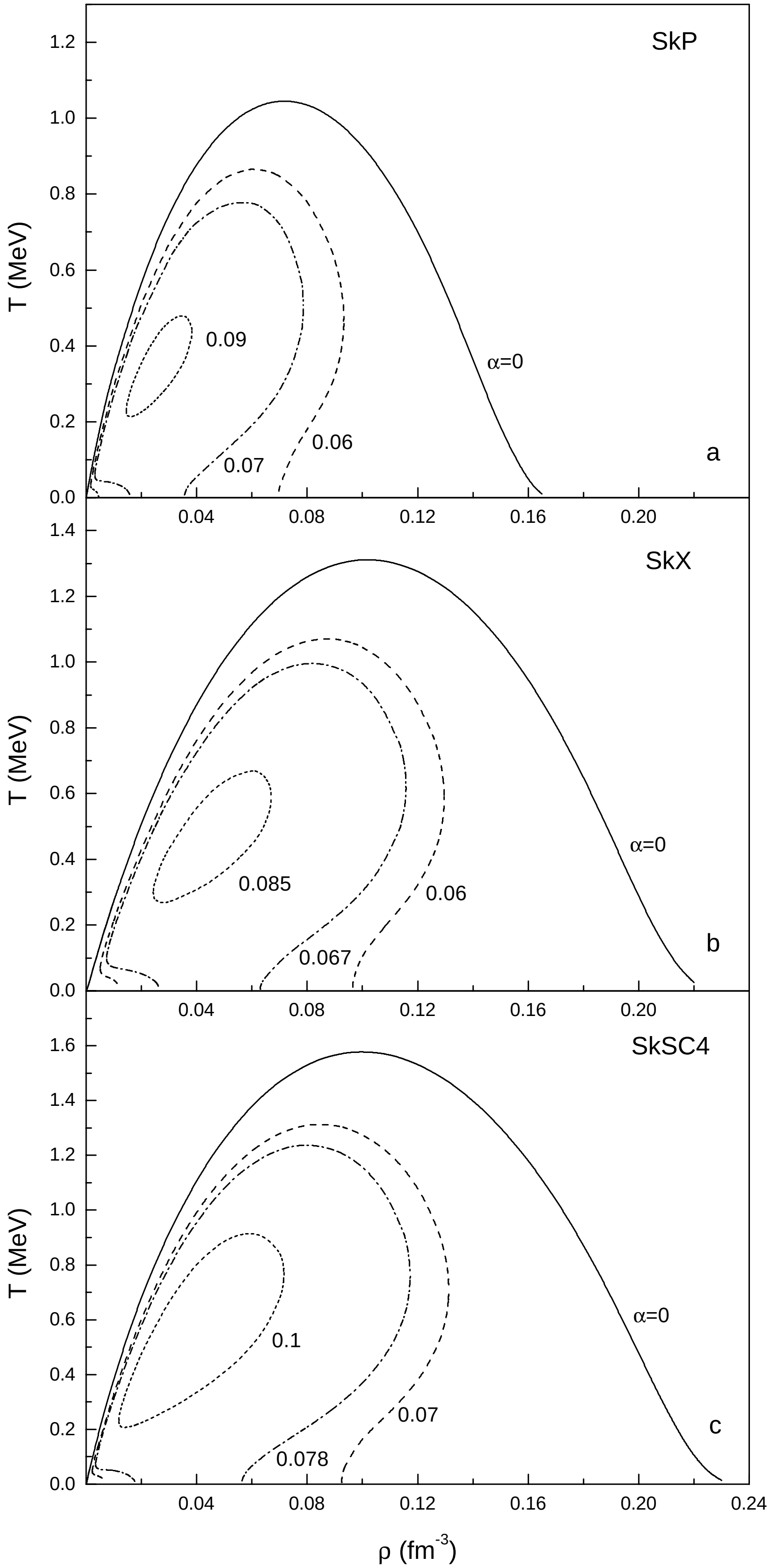

Figure 1: Critical temperature vs density of asymmetric nuclear matter for (a) SkP potential, (b) SkX potential, (c) SkSC4 potential.

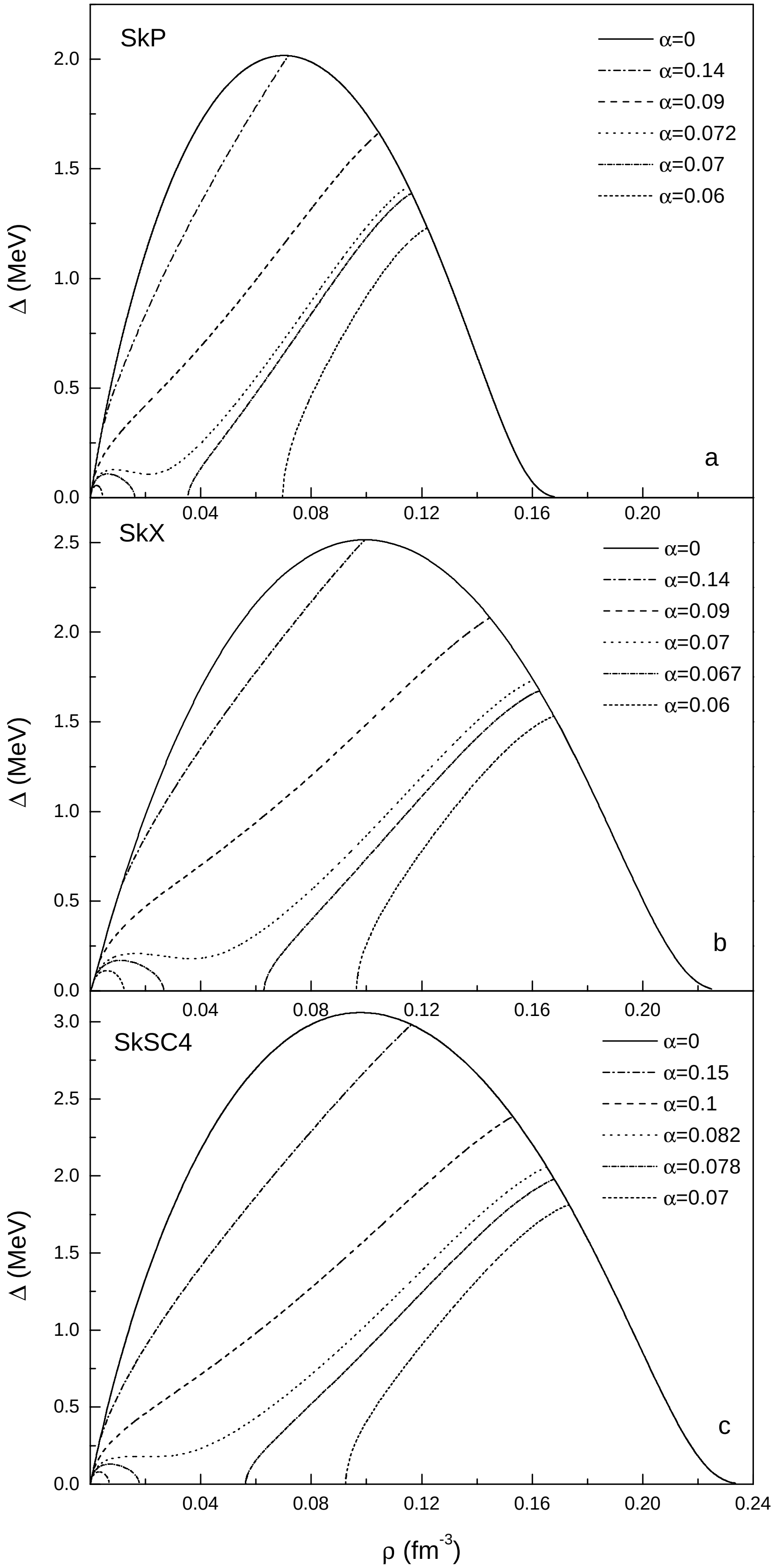

Figure 2: Energy gap at $T = 0$ vs density of asymmetric nuclear matter for (a) SkP potential, (b) SkX potential, (c) SkSC4 potential.

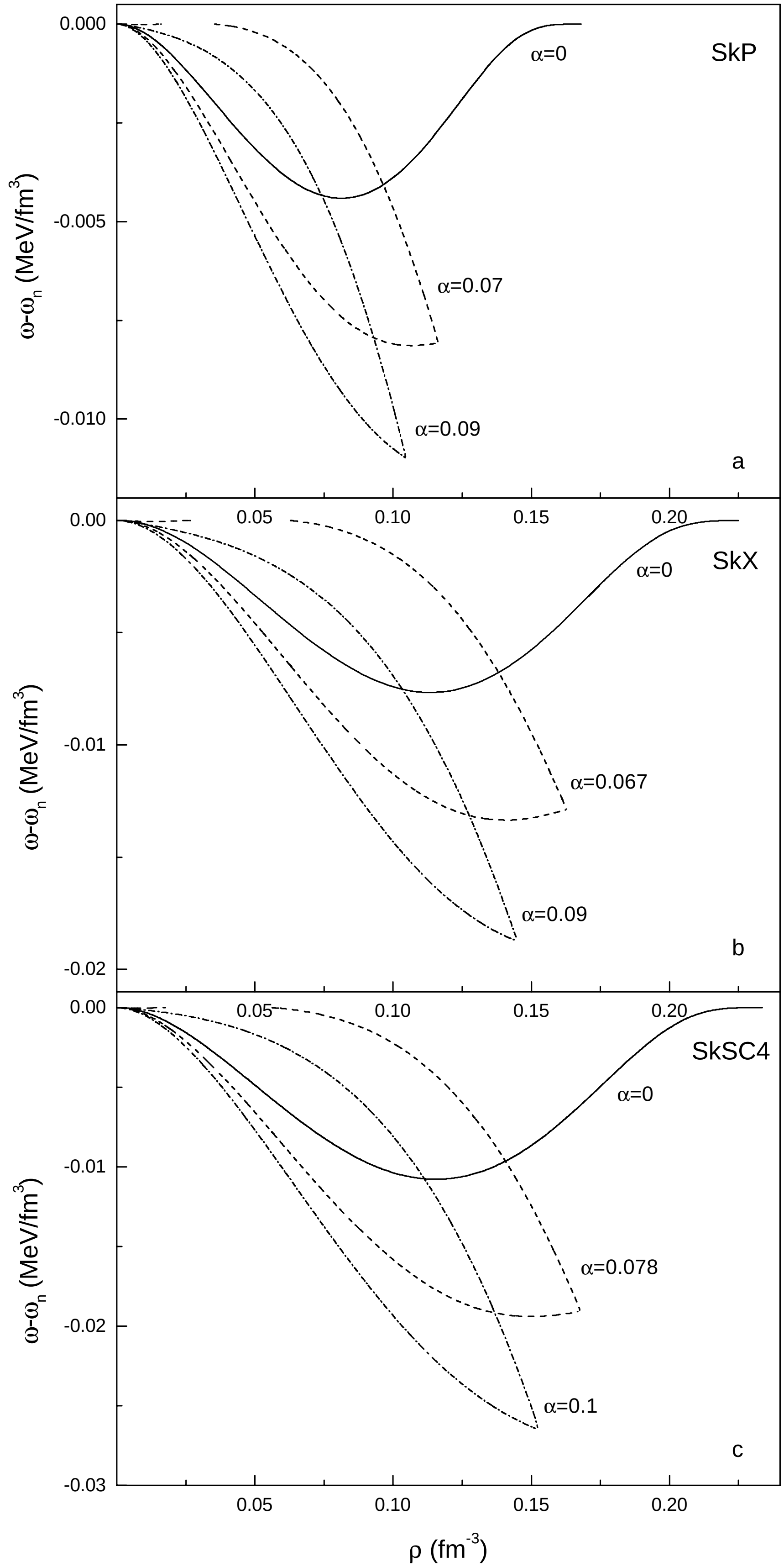

Figure 3: Density of thermodynamic potential, measured from the potential of the normal state, as a function of density for various solutions of self–consistent equation at $T = 0$.